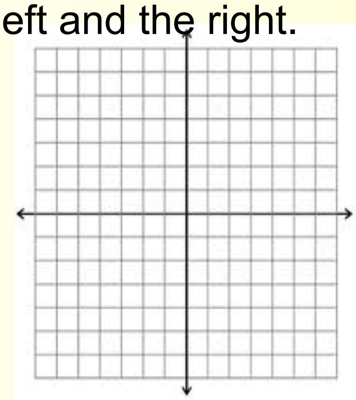
Calculus Warm Up # 1- 4

1. Write the equation of the line in point slope form through: (5, 7) and (-2, 3)

2. Sketch the graph and find the limit as x approaches zero from both the left and the right.

$$f(x) = \begin{cases} 2x - 1, & x < 0 \\ 2x + 3, & x > 0 \end{cases}$$



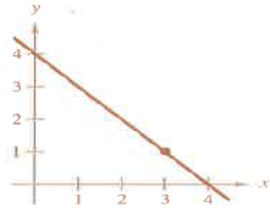
HW ?'s: p. 61 #5, 7-12, 19, 25, 27, 29, 31

5. $\lim_{x \to 3} \frac{[1/(x+1)] - (1/4)}{x - 3}$

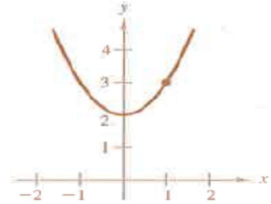
x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$						

In Exercises 7–12, use the given graph to find the limit (if it exists).

7. $\lim_{x \to 3} (4 - x)$

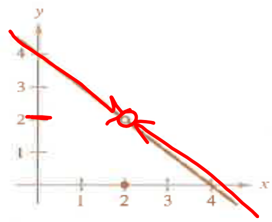


8. $\lim_{x \to 1} (x^2 + 2)$



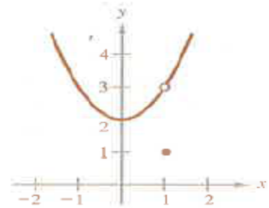
9. $\lim_{x \to 2} f(x) = 2$

$f(x) = \begin{cases} 4 - x, & x \neq 2 \\ 0, & x = 2 \end{cases}$



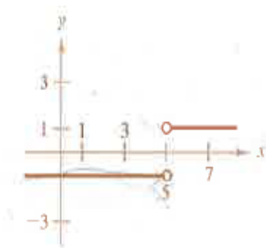
10. $\lim_{x \to 1} f(x)$

$f(x) = \begin{cases} x^2 + 2, & x \neq 1 \\ 1, & x = 1 \end{cases}$

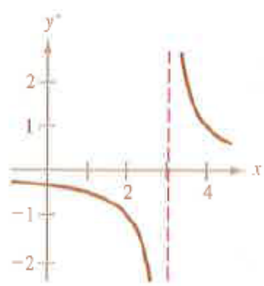


$f(2) = 0$

11. $\lim_{x \to 5} \frac{|x - 5|}{x - 5}$



12. $\lim_{x \to 3} \frac{1}{x - 3}$



Find the limit:

19. $\lim_{x \to 3} \sqrt{x + 1}$

25. $\lim_{x \to -1} \frac{x^2 + 1}{x}$

27. If $\lim_{x \to c} f(x) = 2$ and $\lim_{x \to c} g(x) = 3$, find the following.

(a) $\lim_{x \to c} [5g(x)]$ (b) $\lim_{x \to c} [f(x) + g(x)]$

(c) $\lim_{x \to c} [f(x)g(x)]$ (d) $\lim_{x \to c} \dfrac{f(x)}{g(x)}$

$= \left(\lim_{x \to c} f(x)\right)\left(\lim_{x \to} g(x)\right)$

$= (2)(3) = \boxed{6}$

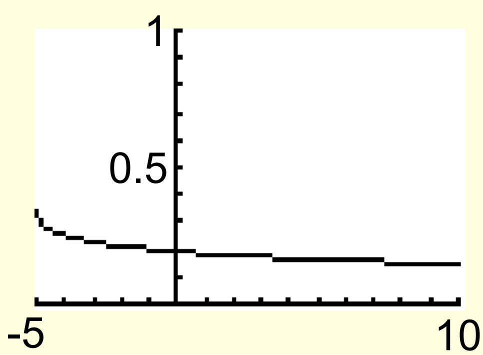
29. If $\lim_{x \to c} f(x) = 4$, find the following.

(a) $\lim_{x \to c} [f(x)]^3$ (b) $\lim_{x \to c} \sqrt{f(x)}$

(c) $\lim_{x \to c} [3f(x)]$ (d) $\lim_{x \to c} [f(x)]^{3/2}$

In Exercises 31 and 32, use a computer or graphics calculator to sketch the graph of the function f and find the specified limit (if it exists).

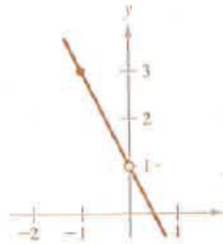
31. $f(x) = \dfrac{\sqrt{x+5} - 3}{x - 4}$, $\lim_{x \to 4} f(x)$



X	Y1
3.92	.16704
3.94	.16695
3.96	.16685
3.98	.16676
4	ERROR
4.02	.16657
4.04	.16648

p. 68 #1-9 odds, 17-23 odds

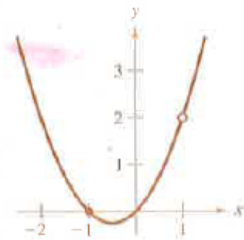
1. $g(x) = \dfrac{-2x^2 + x}{x}$



(a) $\lim_{x \to 0} g(x)$

(b) $\lim_{x \to -1} g(x)$

3. $g(x) = \dfrac{x^3 - x}{x - 1}$



(a) $\lim_{x \to 1} g(x)$

(b) $\lim_{x \to -1} g(x)$

5. $\lim_{x \to -1} \dfrac{x^2 - 1}{x + 1}$

7. $\lim_{x \to 3} \dfrac{x - 3}{x^2 - 9}$

9. $\lim_{x \to -2} \dfrac{x^3 + 8}{x + 2}$

17. $\lim_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 1}$

$$\lim_{x \to -2} \frac{\cancel{(x+2)}(x^2 - 2x + 4)}{\cancel{(x+2)}}$$

19. $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x} \cdot \frac{\sqrt{3+x}+\sqrt{3}}{\sqrt{3+x}+\sqrt{3}}$

$= \lim_{x\to 0} \frac{3+x-3}{x(\sqrt{3+x}+\sqrt{3})}$

$= \lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{3+x}+\sqrt{3})}$

$= \lim_{x\to 0} \frac{1}{\sqrt{3+x}+\sqrt{3}}$

$= \frac{1}{2\sqrt{3}}$

$= \frac{\sqrt{3}}{6}$

21. $\lim_{x\to 0} \frac{[1/(2+x)]-(1/2)}{x}$

$= \lim_{x\to 0} \frac{1}{x}\left[\frac{2}{2}\cdot\frac{1}{2+x} - \frac{1}{2}\frac{(2+x)}{(2+x)}\right]$

$= \lim_{x\to 0} \frac{1}{x}\left[\frac{2-2-x}{2(x+2)}\right]$

$= \lim_{x\to 0} \frac{-\cancel{x}}{\cancel{x}(2x+4)}$

$= \lim_{x\to 0} \left[-\frac{1}{2x+4}\right]$

$= -\frac{1}{2(0)+4}$

$= -\frac{1}{4}$

23. $\lim_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x} \cdot \frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}$

$= \lim_{x\to 0} \frac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})}$

$= \lim_{x\to 0} \frac{1}{\sqrt{x+2}+\sqrt{2}}$

$= \frac{1}{\sqrt{0+2}+\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}}{4}$

Strategies to find limits

1. Look at a graph

2. Look at a table

3. Direct substitution if the function is continuous

4. Factor and cancel to create a new function

5. Rationalize the numerator or denominator

6. Find a common denominator to simplify compound fractions

Today: 2.1 - 2.2 part 2

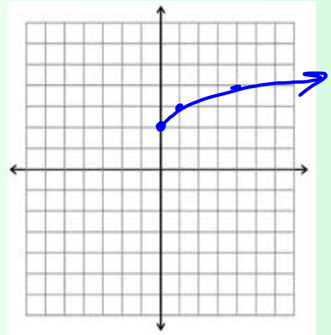
-Finding one-sided limits

-Determining limits that are not numeric values

-Finding the equation of a tangent line at a point on a curve

One-sided limits

Example: $f(x) = \sqrt{x} + 2$



$\lim\limits_{x \to 0^+} f(x) = 2$

The limit does not exist unless we only look at one side.

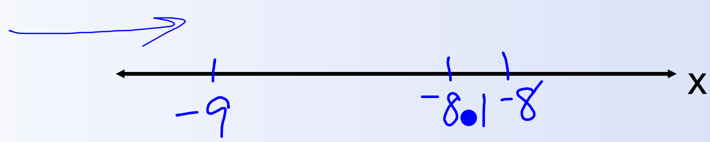
$$19. \lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$$

Another use for one-sided limits:

p. 67

$f(x) = [\![x]\!]$ is called the "Greatest Integer Function"

The outcome, $f(x)$, is the greatest integer value without going over the input, x.



Find:

a. $f(3) = 3$

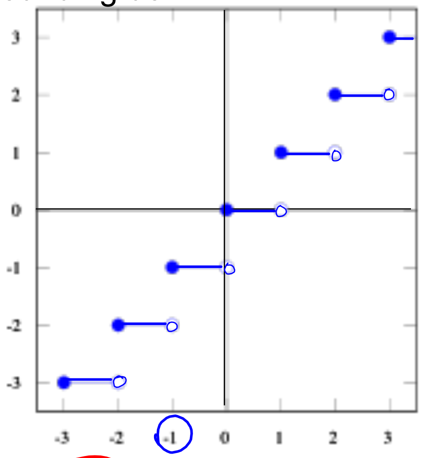
b. $f(7.8) = 7$

c. $f(-8.1) = -9$

d. $f(-15.9) = -16$

$f(x) = [\![x]\!]$

The greatest integer function, rounding down.

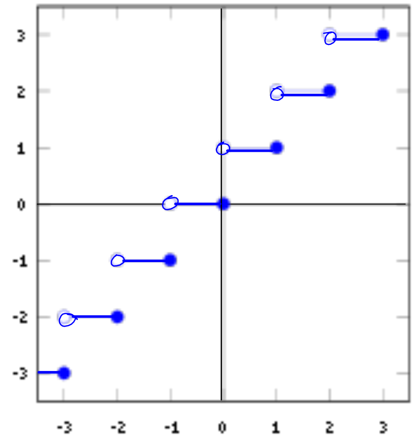


Dom: $\mathbb{R}$

Range: $\mathbb{Z}$

$f(x) = \lceil\!\lceil x \rceil\!\rceil$

The ceiling function, rounding up.



Dom: $\mathbb{R}$

Range: $\mathbb{Z}$

Limits that are not numeric values.
Use the simplifying tools from yesterday!

1. $$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 + (x+\Delta x) - (x^2+x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left(x^2 + 2x\Delta x + (\Delta x)^2 + x + \Delta x - x^2 - x\right)$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left((\Delta x)^2 + 2x\Delta x + \Delta x\right)$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(\Delta x + 2x + 1)}{\cancel{\Delta x}}$$

$$= 0 + 2x + 1$$

$$= 2x + 1$$

2. $$\lim_{y \to 0} \frac{\dfrac{1}{(x+y)^2} - \dfrac{1}{x^2}}{y}$$

2. $\lim_{y\to 0} \dfrac{\dfrac{1}{(x+y)^2} - \dfrac{1}{x^2}}{y}$

$= \lim_{y\to 0} \frac{1}{y}\left[\frac{x^2}{x^2}\frac{1}{(x+y)^2} - \frac{1}{x^2}\frac{(x+y)^2}{(x+y)^2}\right]$

$= \lim_{y\to 0} \frac{1}{y}\left[\frac{x^2-(x+y)^2}{x^2(x+y)^2}\right]$

$= \lim_{y\to 0} \frac{1}{y}\left[\frac{x^2-(x^2+2xy+y^2)}{x^2(x+y)^2}\right]$

$= \lim_{y\to 0} \frac{1}{y}\left[\frac{-2xy-y^2}{x^2(x+y)^2}\right]$

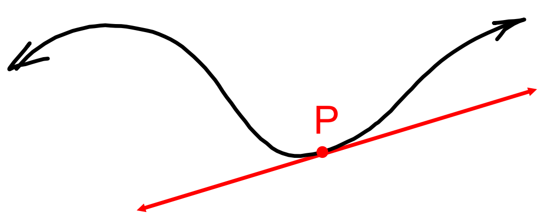
$= \lim_{y\to 0} \frac{-2x-y}{x^2(x+y)^2}$

$= \frac{-2x-0}{x^2(x+0)^2}$

$= \frac{-2x}{x^4}$

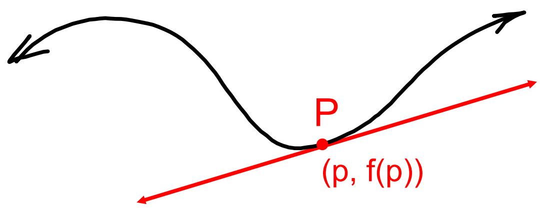
$= \frac{-2}{x^3}$

Finding the equation of a line tangent to a curve.



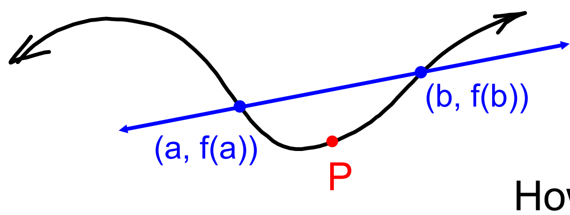
What do we need? Slope & a point

Finding the equation of a line tangent to a curve.



Point - Slope Form

$y - f(p) = m(x - p)$



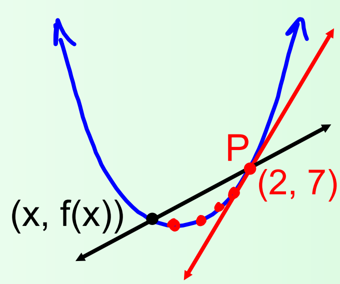
$$\text{slope} = \frac{f(b) - f(a)}{b - a}$$

$\lim_{x \to p}$

How can we use limits to find the slope at P?

Find the slope of f(x) at the point (2, 7)

$$f(x) = x^2 + 3$$



$$m = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$\lim_{x \to 2} \frac{x^2 + 3 - 7}{x - 2}$$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$$

What is the equation of the tangent line?

$m = 4$ $\qquad y - 7 = 4(x - 2)$

HW: p. 69 #33 - 47 odds.

and finish up your function classwork from yesterday.